Singapore vs. Shanghai: A Comparison between primary-level Maths Syllabus and Textbooks

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1. Introduction

As the countries in the Asia-Pacific region experience rapid economic growth, they are gradually producing admirable achievements in the education sector. In the recent 2009 Programme for International Student Assessment (PISA), with the exception of Finland, countries or cities in the Asia-Pacific region have bagged the top five spots in each category. Singapore, Hong Kong and Shanghai in particular, have clinched the top five positions in each of the three categories. Shanghai especially, produced astounding results, topping each of the three categories, listed in Figure 1 below.

Reading		Math		Science	
Country	**Mean**	**Country**	**Mean**	**Country**	**Mean**
Shanghai - China	556	Shanghai - China	600	Shanghai - China	575
South Korea	539	Singapore	562	Finland	554
Finland	536	Hong Kong - China	555	Hong Kong - China	549
Hong Kong - China	533	South Korea	546	Singapore	542
Singapore	526	Chinese Taipei	543	Japan	539

Figure 1: 2009 PISA rankings [1]

A more in-depth comparison reveals Shanghai's superiority in Math, as illustrated in Figure 2:

Country or City	Percentage of elite students		
	Reading	**Math**	**Science**
Singapore	2.6% （5th overall）	15.6% （2nd overall）	4.6% （4th overall）
Shanghai	2.4% （1st overall）	26.6% （1st overall）	3.9% （1st overall）

Figure 2: Comparison of the top students [2]

In terms of the percentage of students in the top group, Singapore has surpassed Shanghai in the categories of Reading and Science even though Singapore's overall ranking is lower. This goes to show that the cultivating of elite students in Singapore is successful. However, in the area of Math, the disparity in the percentage of students in the top group is a substantial 11% even though Singapore ranks second to Shanghai. This huge disparity between the first two cities indicates that Shanghai has far outpaced other countries in the teaching of Math.

Although PISA only assesses secondary students, it ought to also be an accurate reflection of the standard of Math at the primary levels because primary-level Math develops fundamental skills. This essay aims to compare in detail the differences between the syllabi and textbooks of primary-level Math of the two cities and look out for some practices that Singapore could adopt or take reference from Shanghai's pedagogy. Please understand and correct any omissions or discrepancies.

2. Comparison of syllabi

The syllabus is the main guideline of the curriculum. Hence, this essay will make a comprehensive comparison of the syllabi of the two cities in terms of overall design and specific topics. For ease of comparison, this essay utilises the form of a table.

Topic	Shanghai Syllabus [3]	Singapore Syllabus [4]	Remarks
Target audience	A structured curriculum is planned from the primary to the junior college levels.	A structure curriculum is planned only for primary-level Math.	The Singapore secondary and JC syllabi follows the Cambridge 'O' and 'A' levels
Scope	The syllabus has suggestions for all the steps involved in implementation, for instance the prescription of content knowledge, writing of materials, assessment of results and professional development of teachers.	Focuses primarily on the prescription of content knowledge.	
Learning objectives	The mastery of basic concepts, skills, a mathematical way of thinking, logical deduction and application of Mathematical concepts to analyse and solve problems, and most importantly the ability to clearly and concisely demonstrate the thinking process.	Requires step-by-step working in daily assignments, but does not explicitly state such a requirement in the syllabus.	One of the most important goals of learning Math ought to be the ability to express clearly and concisely the Mathematical language.
Numbers and operations	1. P1 (Sem 1): Students learn the meaning and use of symbols ">" and "<". 2. P3 (Sem 2): Students learn to use the calculator. 3. P4 (Sem 1): Students learn the rules of operation like commutative, associative and distributive laws. 4. P5 (Sem 1): Students learn multiplication and division with decimals.	1. The syllabus specifically excludes the symbols ">" and "<" but teaches the concept in words. 2. P5 (Sem 1): Students learn to use the calculator. 3. Absence of the rules of operation. 4. Absence of multiplication and division using decimals.	Students from Shanghai are introduced to the calculator earlier than Singaporean peers, but it does not appear to affect their powers of computation. This might be due to Shanghai's emphasis on calculation skills.
Measurements	1. Length – covers all the units of measurement, including decimetres. 2. Time – In addition to calculations on a timeline, students also learn the addition and subtraction of time.	1. Length – Does not cover the decimetre, which affects the expression of the unit of measurement of volume $\ell = dm^3$ 2. Time – All calculations are performed on the timeline.	The Singapore syllabus seems to over-simplify due to an overt concern with students' ability to calculate.
Percentages	P6 (Sem 1): Students start learning about percentages, with special focus on teaching it as a kind of fraction with a denominator of 100.	P5 (Sem 2): Students start learning percentages with no link between percentages and fractions. There was a question in a textbook [5] to express 17/100 as a percentage (17%) when it is already one.	This could be due to the problem of textbooks, not the syllabus

Geometry	There is an emphasis on students' drawing and mapping.	The students' drawing and mapping skills are inadequate.	In geometry, students need to take pen to paper to draw figures themselves as part of the learning process.
Data analysis	1. Less time is spent on this topic, only during P2 (Sem 2), P4 (Sem 2) and P5 (Sem 1). 2. There is an emphasis on the conversion from tables to graphs. 3. There is no mention of pie charts.	1. There is more time spent on this topic starting from P1 Sem 2 and in the second semester of every year after. 2. There is an emphasis on knowing and applying, with no mention about the conversion from tables to graphs. 3. Pie charts are introduced in P6 (Sem 2).	1. Tables must necessarily come before graphs. 2. Pie charts are worth learning.
Algebra and Equations	1. P5 (Sem 1): Students start to learn algebra and equations. 2. Students start to learn the concepts and symbols of positive and negative numbers in order to facilitate the computation of algebraic equations.	1. P6 (Sem 1): Students learn about algebra but not equations. 2. There is no mention of the concept (and symbols) of positive and negative numbers.	Several decades ago, Shanghai did not teach algebra and equations to primary school students in order to train logical thinking. However, the questions the Shanghai Primary 6 students deal with far surpass those set decades ago in terms of difficulty. Algebra and equations hence become a necessity. In fact, even in Singapore a sample question in a Primary 2 assessment book posts a rudimentary equation: ♥+♥=4.

3. Comparison of textbooks

The content of textbooks in Singapore [5] and Shanghai [6] accurately reflect the requirements of the syllabi, and both are commendable. Listed below are some salient features in Shanghai's textbooks that are worth recommending.

(1) Chapters reflect an actual understanding of the process

The content and arrangement of each chapter reflects the actual understanding of the process. For instance, when teaching graphs and tables, tables come first, emphasizing the conversion from tables to graphs before proceeding. In addition, textbooks state explicitly the formal requirements of graphs.

(2) Chapters emphasise the application of knowledge

Each chapter leads in with one or several real life examples. In addition, the arrangement of content also reflects application. For example, questions regarding speed form an important chapter in Singapore's Primary 6 syllabus. However, Shanghai incorporates these questions into their chapter of the application of algebra and equations.

(3) Chapters emphasise links between topics and links between summaries

Shanghai textbooks clearly list tables, bar graphs, line graphs and averages under the topic of data analysis. There is heavy emphasis on the links between the individual topics. For example, the title of the topic in Shanghai is "Data Analysis – Averages" instead of "Averages". The syllabus in Singapore explicitly classifies the topics, but the textbooks do not have this express classification [5].

Another point worthy of mention is a summary after chapters which are interlinked. For example, in a typical Primary 5 textbook, the chapter "Data Analysis – Averages" ends with a summary of all the topics (tables, bar graphs and line graphs) within the same broad category of data analysis even though the topics were taught in previous years.

(4) Textbooks incorporate Mathematical Olympiad skills

Both the ministries of education in Singapore and Shanghai oppose Mathematical Olympiad competitions, but Mathematical Olympiad skills can be categorized into two levels- basic and competitive. The learning of basic Mathematical Olympiad skills is helpful to students in cultivating logical thinking and creative thinking. Shanghai textbooks selectively include a number of Mathematical Olympiad skills.

(5) Textbooks incorporate auxiliary computing tools and stories about Chinese mathematicians

The teaching materials from Shanghai incorporate helpful computing tools and games in order to help students learn better. The materials also include short introductions to mathematicians and brief historical accounts to pique their interest in Mathematics. For example, the history of Chinese mathematician Zu Chongzhi and his discovery of *pi*, the ratio of the circumference of a circle to its diameter, accurate to seven decimal places are recounted.

4. Summary and Recommendations

The syllabi and textbooks of Singapore and Shanghai have different strengths and would benefit from learning from each other. The syllabus is the mainstay on which teaching materials and every other aspect is based. A good syllabus should reflect both the nature of learning Math and the current standard of teaching.

The nature of learning Math is to train logical thinking and expression in Mathematical terms. Learning should allow us to express a practical problem in terms of a Math question, and after logical thinking, apply Math concepts and skills to analyse and solve the problem, arriving at real-life application.

The difficulty of practice questions and examinations set in schools has far exceeded the syllabi and teaching materials. It is imperative that we redefine the syllabus and textbooks in order to accurately reflect the nature and current level of Math.

Citations

[1] *PISA 2009 Results: Executive Summary*, www.pisa.oecd.org

[2] 陆璟等，《PISA 对考试和课程改革的启示》，（上海市教科院普教所，2011）

[3] 《上海市中小学数学课程标准》，2004

[4] *Mathematics Syllabus Primary*, Singapore Ministry of Education, http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2007.pdf

[5] *My Pals Are Here - Maths 2nd Edition (P1-P6)*, Marshall Cavendish, 2007

[6] 《数学课本》（一年级 – 六年级），（上海教育出版社，2004）